Federal State Autonomous Educational Institution of Higher Education "Moscow Institute of Physics and Technology (National Research University)"

APPROVED
Vice Rector for Academic Affairs

A.A. Voronov

Work program of the course (training module)

course:	Mathematical Analysis – Functions of One Variable/Математический анализ – функции одной переменной
major:	Biotechnology
specialization:	Biomedical Engineering/Биомедицинская инженерия
	Phystech School of Biological and Medical Physics
	Chair of Higher Mathematics
term:	1
qualification:	Bachelor

Semester, form of interim assessment: 2 (spring) - Exam

Academic hours: 120 AH in total, including:
lectures: 60 AH.
seminars: 60 AH.
laboratory practical: 0 AH.

Independent work: 120 AH.

Exam preparation: 30 AH.

In total: 270 AH, credits in total: 6

Number of course papers, tasks: 3

Authors of the program:

G.E. Ivanov, doctor of physics and mathematical sciences, full professor, head of chair
L.N. Znamenskaya, doctor of physics and mathematical sciences, full professor, professor
O.V. Besov, doctor of physics and mathematical sciences, full professor, professor
A.Y. Petrovich, candidate of physics and mathematical sciences, associate professor, associate professor
O.G. Podlipskaya, candidate of physics and mathematical sciences, associate professor, associate professor
O.E. Orel, candidate of physics and mathematical sciences, associate professor, associate professor

The program was discussed at the Chair of Higher Mathematics 20.05.2021

Annotation

Discipline belongs to the basic part of the educational program. Mastering the discipline is aimed at developing the ability to acquire new scientific and professional knowledge using modern educational and information technologies. Topics covered include Derivative and its applications, Higher order derivatives, Antiderivative and indefinite integral, Differential geometry, Defined integral, Improper integral.

1. Study objective

Purpose of the course

Formation of basic knowledge in mathematical analysis for further use in other areas of mathematical knowledge and disciplines with natural science content; the formation of a mathematical culture, research skills and the ability to apply knowledge in practice.

Tasks of the course

- Acquisition of theoretical knowledge and practical skills by students in the field of the theory of limits, differential and integral calculus, the theory of series;
- preparing students for the study of related mathematical disciplines;
- the acquisition of skills in the application of methods of mathematical analysis in physics and other natural science disciplines.

2. List of the planned results of the course (training module), correlated with the planned results of the mastering the educational program

Mastering the discipline is aimed at the formation of the following competencies:

Code and the name of the competence	Competency indicators
UC-1 Search and identify, critically assess, and synthesize information, apply a systematic approach to problem-solving	UC-1.1 Analyze problems, highlight the stages of their solution, plan the actions required to solve them
	UC-1.2 Find, critically assess, and select information required for the task in hand
	UC-1.3 Consider various options for solving a problem, assess the advantages and disadvantages of each option
	UC-1.4 Make competent judgments and estimates supported by logic and reasoning
UC-6 Use time-management skills, apply principles of self-development and lifelong learning	UC-6.2 Plan independent activities in professional problem-solving; critically analyze the work performed; find creative ways to use relevant experience for self-development

3. List of the planned results of the course (training module)

As a result of studying the course the student should:

know:

- basic properties of the limits of sequences and functions of a real variable, derivative, differential, indefinite integral; properties of functions that are continuous on a segment;
- basic "remarkable limits", tabular formulas for derivatives and indefinite integrals, differentiation formulas, basic expansions of elementary functions according to Taylor's formula;
- basic formulas of differential geometry;
- definition and properties of definite and indefinite integrals, their relationship;
- convergence criteria for improper integrals with power, logarithmic and exponential singularities.

be able to:

- write down statements using logical symbols;
- calculate the limits of sequences and functions of a real variable;
- calculate the derivatives of elementary functions, expand elementary functions according to the Taylor formula; calculate the limits of functions using the Taylor formula and L'Hôpital's rule;
- build graphs of functions using the first and second derivatives; explore functions for local extremum, as well as find their largest and smallest values at intervals;
- calculate the curvature of plane and spatial curves;
- calculate the indefinite and definite integral;
- use a certain integral to solve applied problems.

master:

- the subject language of classical mathematical analysis, used in the construction of the theory of limits;
- the apparatus of the theory of limits, differential and integral calculus for solving various problems arising in physics, technology, economics and other applied disciplines.

4. Content of the course (training module), structured by topics (sections), indicating the number of allocated academic hours and types of training sessions

4.1. The sections of the course (training module) and the complexity of the types of training sessions

№	Topic (section) of the course	Types of training sessions, including independent work			
		Lectures	Seminars	Laboratory practical	Independent work
1	Taylor's formula	10	10		16
2	Application of the derivative to the study of functions	6	6		14
3	Differential geometry elements	10	10		20
4	Indefinite integral	4	4		8
5	Definite integral	4	4		14
6	Improper integral	10	10		16
7	Derivatives of functions	10	10		16
8	Number series	6	6		16
AH in total		60	60		120
Exam preparation		30 AH.			
Total complexity		270 AH., credits in total 6			

4.2. Content of the course (training module), structured by topics (sections)

Semester: 2 (Spring)

1. Taylor's formula

Basic rules for differentiation. Fermat's theorem (a necessary condition for a local extremum). Rolle, Lagrange, Cauchy mean theorems.

Derivative of an inverse and complex function.

Higher order derivatives. Leibniz's formula for the n-th derivative of the product. Differential of the second order. The lack of invariance of its form with respect to the change of variable. Higher-order differentials.

2. Application of the derivative to the study of functions

Taylor's formula with remainder in the Peano and Lagrange forms.

L'Hôpital's rule for disclosing species uncertainties.

3. Differential geometry elements

Sufficient conditions for monotonicity, sufficient conditions for a local extremum in terms of the first and second derivatives. Convexity, inflection points. Sufficient conditions for a local extremum in terms of higher derivatives.

Plotting functions - asymptotes, studying the intervals of monotonicity and points of local extremum, intervals of convexity and points of inflection. Study for convexity and concavity. Plotting functions.

4. Indefinite integral

Curves on the plane and in space. Smooth curves tangent to a smooth curve.

Lagrange's theorem for vector functions, arc length of a curve.

Derivative of variable arc length. Natural parameter. Curvature of a curve, formulas for its calculation. Accompanying trihedron of the space curve.

5. Definite integral

Antiderivative and indefinite integral. Linearity of the indefinite integral, integration by substitution and by parts. Integration of rational functions.

Basic techniques for integrating irrational and transcendental functions.

6. Improper integral

The definite Riemann integral. Riemann sums, Darboux sums, integrability criterion.

Integrability of a continuous function, integrability of a monotone function, integrability of a bounded function with a finite number of discontinuity points.

Properties of integrable functions: additivity of an integral over segments, linearity of an integral, integrability of a product, integrability of the modulus of an integrable function, integration of inequalities, mean value theorem.

Integral properties with variable upper limit - continuity, differentiability. Newton-Leibniz formula. Integration by substitution and by parts in a definite integral.

Geometric applications of a definite integral - area of a curved trapezoid, volume of a body of revolution, length of a curve, surface area of revolution.

7. Derivatives of functions

Improper integral (the case of an unbounded function and the case of an infinite limit of integration). Cauchy's criterion for the convergence of an integral.

Integrals of functions of constant sign, criteria for comparing convergence. Integrals of alternating functions; absolute and conditional convergence. Dirichlet and Abel signs.

8. Number series

Number series. Cauchy's criterion for convergence of a series. Sign-constant series: criteria for comparing convergence, d'Alembert and Cauchy criteria, integral criterion. Alternating series: absolute and conditional convergence. Dirichlet and Abel signs.

The independence of the sum of an absolutely convergent series on the order of the terms. Riemann's theorem on the permutation of the terms of a conditionally convergent series. The product of absolutely converging series.

5. Description of the material and technical facilities that are necessary for the implementation of the educational process of the course (training module)

Classroom equipped with a multimedia projector, screen and microphone.

6. List of the main and additional literature, that is necessary for the course (training module) mastering

Main literature

1. Advanced calculus, A. Friedman ; The Ohio State University. Mineola ; New York, Dover publications, inc., 2016

2. Mathematical analysis II /V. A. Zorich. Berlin, Springer, 2016

Additional literature

1. Лекции по математическому анализу [Текст] : [в 2 ч.] : учеб. пособие для вузов. Ч. 1 / Г. Н. Яковлев .— 2-е изд., перераб. и доп. — М. : Физматлит, 2004 .— 340 с.

Кудрявцев, Л. Д.

Краткий курс математического анализа [Текст]. В 2 т. Т. 1. Дифференциальное и интегральное исчисления функций одной переменной. Ряды : учеб. пособие для вузов / Л. Д. Кудрявцев .— 3-е изд., перераб. — М. : Физматлит, 2008, 2009 .— 400 с. - Электронная копия доступна на сайте электронно-библиотечной системы. - Предм. указ.: с. 396-399. - 1000 экз. - ISBN 978-5-9221-0184-4 (в пер.) .— URL: https://e.lanbook.com/book/2224 (дата обращения: 29.12.2020). - Полный текст (Режим доступа : из сети МФТИ / Удаленный доступ).

7. List of web resources that are necessary for the course (training module) mastering

http://www.math.mipt.ru

8. List of information technologies used for implementation of the educational process, including a list of software and information reference systems (if necessary)

The lectures use multimedia technologies, including the demonstration of presentations.

9. Guidelines for students to master the course

Provided in the annually developed homework assignments.

SUPPLEMENT

Assessment funds for course (training module)

major: Biotechnology
specialization: Biomedical Engineering/Биомедицинская инженерия
Phystech School of Biological and Medical Physics
Chair of Higher Mathematics
term: 1
qualification: Bachelor

Semester, form of interim assessment: 2 (spring) - Exam

Authors:

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O.G. Podlipskaya, candidate of physics and mathematical sciences, associate professor, associate professor
O.E. Orel, candidate of physics and mathematical sciences, associate professor, associate professor

1. Competencies formed during the process of studying the course

Code and the name of the competence	Competency indicators
UC-1 Search and identify, critically assess, and synthesize information, apply a systematic approach to problem-solving	UC-1.1 Analyze problems, highlight the stages of their solution, plan the actions required to solve them
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	UC-1.4 Make competent judgments and estimates supported by logic and reasoning
UC-6 Use time-management skills, apply principles of self-development and lifelong learning	UC-6.2 Plan independent activities in professional problem-solving; critically analyze the work performed; find creative ways to use relevant experience for self-development

2. Competency assessment indicators

As a result of studying the course the student should:

know:
- basic properties of the limits of sequences and functions of a real variable, derivative, differential, indefinite integral; properties of functions that are continuous on a segment;
- basic "remarkable limits", tabular formulas for derivatives and indefinite integrals, differentiation formulas, basic expansions of elementary functions according to Taylor's formula;
- basic formulas of differential geometry;
- definition and properties of definite and indefinite integrals, their relationship;
- convergence criteria for improper integrals with power, logarithmic and exponential singularities.

be able to:
- write down statements using logical symbols;
- calculate the limits of sequences and functions of a real variable;
- calculate the derivatives of elementary functions, expand elementary functions according to the Taylor formula; calculate the limits of functions using the Taylor formula and L'Hôpital's rule;
- build graphs of functions using the first and second derivatives; explore functions for local extremum, as well as find their largest and smallest values at intervals;
- calculate the curvature of plane and spatial curves;
- calculate the indefinite and definite integral;
- use a certain integral to solve applied problems.

master:
- the subject language of classical mathematical analysis, used in the construction of the theory of limits;
- the apparatus of the theory of limits, differential and integral calculus for solving various problems arising in physics, technology, economics and other applied disciplines.

3. List of typical control tasks used to evaluate knowledge and skills

Current control is carried out on the basis of a point-rating system (BRS) for evaluating knowledge in the discipline being studied. The BRS takes into account the students ' performance of a set of homework assignments and tests in accordance with the curriculum. Data on attendance and current academic performance are entered by teachers in special journals and recorded in the BRS.

Current control on the basis of homework is carried out during the academic semester in the terms set by the Educational Department, in accordance with the curriculum.
To pass the task, the student must provide a solution to the homework problem in writing, answer the questions of the teacher and write a test paper on the task, which checks the knowledge of concepts and statements on the topics of the task and the ability to solve problems.
You can't use other people's help, computers, or mobile phones during the test.

* A BAR is attached to the subject being studied.

4. Evaluation criteria

Certification in the discipline "Mathematical Analysis – Functions of One Variable" is carried out in the form of an exam.
The exam is conducted taking into account the control tasks previously completed by the students.

Control tasks:

1. Derive formulas for the derivative of a complex and inverse function.
2. Formulate and prove the theorems of Rolle, Lagrange and Cauchy.
3. Necessary and sufficient conditions for the extremum of a function of one variable.
4. Derive the Taylor formula with the remainder in the form of Peano and Lagrange.
5. Curve on the plane and in space, the length of the arc of the curve.
6. Give the definition of a definite integral. Prove Darboux's criterion for integrability of functions.
7. Prove the boundedness of a Riemann integrable function.
8. Prove the Riemann integrability of the bounded function.
9. Prove the Newton-Leibniz formula.
10. Give the definition of an improper integral. Which integrals are called conditionally convergent, and which are absolutely convergent?
11. Prove the Cauchy criterion for the convergence of an improper integral.
12. Prove the Dirichlet and Abel criteria for the convergence of an improper integral.
13. Formulate and prove the d'Alembert and Cauchy tests for convergence of a number series with non-negative terms.
14. Prove the Dirichlet and Abel criteria for convergence of a series with arbitrary coefficients.
15. The independence of the sum of an absolutely convergent series from the order of the terms.
16. Riemann's theorem on the permutation of the terms of a conditionally convergent series.
17. Product of absolutely converging series.

Examples of exam tickets:

Ticket 1
1. Prove the Dirichlet and Abel criteria for the convergence of an improper integral.
2. Give the definition of a number series. Prove the Cauchy criterion for the convergence of a number series.

Ticket 2
1. Derive formulas for the derivative of a complex and inverse function.
2. Prove the Riemann integrability of a bounded function. Prove the Riemann integrability of a bounded function.

Grade "excellent (10)" is given to a student who has exhibited extensive and deep knowledge of the course and ability to apply skills when solving specific tasks;

Grade "excellent (9)" is given to a student who has exhibited extensive and deep knowledge of the course and ability to apply skills when solving specific tasks, but he has made minor errors that were independently found and corrected;

Grade "excellent (8)" is given to a student who has exhibited extensive and deep knowledge of the course and ability to apply skills when solving specific tasks, but he has made minor errors that were independently corrected after the instructions of an examiner;

Grade "good (7)" is given to a student who has a good command of the course and is able to apply skills when solving specific tasks, but has made minor mistakes when answering questions or solving problems;

Grade "good (6)" is given to a student who has a good command of the course and is able to apply skills when solving specific tasks, but has made rare mistakes when answering questions or solving problems;

Grade "good (5)" is given to a student who has a good command of the course and is able to apply skills when solving specific tasks, but has made mistakes when answering questions or solving problems;

Grade "satisfactory (4)" is given to a student who has exhibited fragmented knowledge, has made inaccurate formulation of the basic concepts, but understands the subject well, is able to apply the knowledge in standard situations and possesses skills necessary for the future study;

Grade "satisfactory (3)" is given to a student who has exhibited fragmented knowledge, has made inaccurate formulation of the basic concepts, has inconsistencies in understanding the course, but is able to apply the knowledge in standard situations and possesses skills necessary for the future study;

Grade "unsatisfactory (2)" is given to a student who does not possess knowledge of the essential concept of the course, has made gross mistakes in formulations of basic concepts and cannot use the knowledge in solving typical tasks;

Grade "unsatisfactory (1)" is given to a student who has exhibited total lack of knowledge of the course.

5. Methodological materials defining the procedures for the assessment of knowledge, skills, abilities and/or experience

The written examination takes 4 astronomical hours.

When conducting an oral exam, the student is given 1 astronomical hour for preparation. The student's questionnaire on the oral exam must not exceed 2 astronomical hours.

During the exam, students can use only the discipline program.

Scoring for Exams with Written Part
Department of Higher Mathematics

NN	Types of Control	Points
1	Control test 1	0—6
2	Control test 2	0—6
3	Control test 3	0—6
4	Homework 1	0—2
5	Homework 2	0—2
6	Homework 3	0—2
7	Theory checking	0—3
8	Class attendance and activity during seminars	0—3
9	Written exam	0—30
10	Oral exam	0—60
	Total Score	0—120

The score for the oral exam is computed by the formula $6 \cdot N$, where $N \geq 3$ is a grade gained by a student during the exam. If $N = 1$ or 2, then the student grade is equal to N.

Conversion Scale between the total score and the student grade

Total Score	**Student Grade**	
112—120	10	Excellent
103—111	9	
94—102	8	
85—93	7	Good
76—84	6	
67—75	5	
54—66	4	Satisfactory
41—53	3	
27—40	2	Unsatisfactory
0—26	1	